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Eigensensitivity Analysis for Asymmetric Nonviscous Systems

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I. Introduction

THE eigensensitivities of mechanical systems with respect to structural design parameters have become an integral part of many engineering design methodologies including optimization, structural health monitoring, structural reliability, model updating, dynamic modification, reanalysis techniques, and many other applications. Fox and Kapoor [1] computed the derivative of each eigenvector as a linear combination of all of the undamped eigenvectors. Later, Adhikari and Friswell [2] and Adhikari [3] extended the modal method to the more general asymmetric systems with viscous and nonviscous damping, respectively. Nelson [4] presented a method, which requires only the eigenvector of interest by expressing the derivative of each undamped eigenvector as a particular solution and a homogeneous solution. Later, Friswell and Adhikari [5] extended Nelson's method to symmetric and asymmetric systems with viscous damping. Recently, Adhikari and Friswell [6] extended Nelson's method to symmetric and asymmetric nonviscously damped systems. Fox and Kapoor [1] also suggested a direct algebraic method to calculate the eigensensitivity for symmetric undamped systems by solving a nonsingular linear system of algebraic equations. Lee et al. [7] derived an efficient algebraic method, which has a compact linear system with a symmetric coefficient matrix for symmetric systems with viscous damping. Later, Guedria et al. [8] extended the algebraic method to general asymmetric viscous damped systems. Chouchane et al. [9] reviewed the algebraic method and extended their method to the second-order and high-order derivatives of eigensolutions. Li et al. [10] extended the algebraic method to symmetric and asymmetric nonviscously damped systems. Xu and Wu [11] proposed a new normalization and presented a method for the computation of eigensolution derivatives of asymmetric systems with viscously damping. Recently, Mirzaeifar et al. [12] proposed a new method based on a combination of algebraic and modal methods for generally asymmetric viscously damped systems. More recently, Li et al. [13] proposed a method of design sensitivity analysis of asymmetric viscously damped systems with distinct and repeated eigenvalues, which can compute the left and right eigenvector derivatives separately and independently. All of the methods mentioned previously compute the eigensensitivities of asymmetric damped systems by using the left eigenvector. However, these methods have disadvantages in computational cost and storage capacity for the left eigenvector should be calculated.

To avoid using the left eigenvector, an algebraic method is presented [14], which does not require the left eigenvector for asymmetric damped systems, but this method is restricted to the case of viscous damping. It should be noted that the coefficient matrices of the algebraic method may be ill conditioned due to the components of the additional constraints, and system matrices in the coefficient matrices are not all of the same order of magnitude. In addition [15], the normalization adapted in [14] and [12] will fail in some cases because it can equal zero or a very small number. This Note will present a method, which is well conditioned and can calculate the eigensensitivity of asymmetric nonviscous damped systems without using the left eigenvector.

Considering an N-degree-of-freedom linear system with nonviscous (viscoelastic) damping [3,6,10]

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \int_{-\infty}^{t} \mathbf{g}(t-\tau)\dot{\mathbf{u}}(\tau)\,\mathrm{d}\tau + \mathbf{K}\mathbf{u}(t) = \mathbf{0} \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K} \in \mathbb{R}^{N\times N}$ are, respectively, the mass and stiffness matrices, $\mathbf{g}(t) \in \mathbb{R}^{N\times N}$ is the matrix of kernel functions, and $t \in \mathbb{R}^{+}$ denotes time. In the special case, when $\mathbf{g}(t-\tau) = \mathbf{C}\delta(t-\tau)$ where $\mathbf{C}$ is a constant matrix, Eq. (1) reduces to the case of a viscously damped system. Therefore, the nonviscous damping model is considered as a further generalization of the familiar viscous damping. Taking the Laplace transform of the preceding equation gives

$$(s^2\mathbf{M} + s\mathbf{G}(s) + \mathbf{K})\mathbf{u}(s) = \mathbf{0} \quad \text{or} \quad \mathbf{D}(s)\mathbf{u}(s) = \mathbf{0} \tag{2}$$

where $\mathbf{u}(s) = L[\mathbf{u}(t)] \in \mathbb{C}^N$, $\mathbf{G}(s) = L[\mathbf{g}(t)] \in \mathbb{C}^{N\times N}$ and $L[]$ denotes the Laplace transform. The eigenvalues, λ_i, are roots of the characteristic equation

$$\det[s^2\mathbf{M} + s\mathbf{G}(s) + \mathbf{K}] = \mathbf{0} \tag{3}$$

In general, the component of $\mathbf{G}(s)$ can be represented by the following form for the linear viscoelastic case [16]:

$$\mathbf{G}(s) = \frac{p_{jk}(s)}{q_{jk}(s)} \tag{4}$$

where $p_{jk}(s)$ and $q_{jk}(s)$ are finite-order polynomials in s and the degree of $q_{jk}(s)$ is not less than that of $p_{jk}(s)$. Under such assumptions, the order of the characteristic equation m is usually more than $2N$. Therefore, although the system has N degrees of freedom, the number of eigenvalues is more than $2N$. This is a major difference between viscously and nonviscously damped systems. In [17], the eigenvectors can be divided into the elastic modes (corresponding to N complex conjugate pairs of eigenvalues) and

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the nonviscous modes (corresponding to the additional $m-2N$ eigenvalues).

In this Note, the system matrices are asymmetric. The right and left eigenproblems can be expressed, respectively, as follows:

$$(\lambda_i^2\mathbf{M}+\lambda_i\mathbf{G}(\lambda_i)+\mathbf{K})\mathbf{u}_i=\mathbf{0} \quad \text{or} \quad \mathbf{D}(\lambda_i)\mathbf{u}_i=\mathbf{0} \tag{5}$$

$$(\lambda_i^2\mathbf{M}+\lambda_i\mathbf{G}(\lambda_i)+\mathbf{K})^T\mathbf{v}_i=\mathbf{0} \quad \text{or} \quad \mathbf{D}(\lambda_i)^T\mathbf{v}_i=\mathbf{0} \tag{6}$$

where $\mathbf{u}_i$ and $\mathbf{v}_i$ denotes, respectively, the right and left eigenvectors corresponding to the ith eigenvalue λ_i. For the asymmetric matrices, the left and right eigenvectors are not equal, $\mathbf{v}_i \neq \mathbf{u}_i$, therefore, the left eigenvector cannot be obtain from the right eigenvector. We assume that the eigenvalues are distinct. To uniquely determine the eigenvectors, the following normalization is adopted [3,6,10]:

$$\mathbf{v}_i^T\frac{\partial\mathbf{D}(s)}{\partial s}\bigg|_{s=\lambda_i}\mathbf{u}_i=\theta_i \tag{7}$$

where

$$\frac{\partial\mathbf{D}(s)}{\partial s}\bigg|_{s=\lambda_i}=2\lambda_i\mathbf{M}+\mathbf{G}(\lambda_i)+\lambda_i\frac{\partial\mathbf{G}(s)}{\partial s}\bigg|_{s=\lambda_i}$$

Here θ_i is some nonzero constant. Often numerical values of θ_i should be chosen in various ways, hence, this normalization is inconsistent with undamped or classically damped modal theories (the details can be seen in [3]). In addition, this normalization needs both the right and left eigenvector. To avoid using the left eigenvector, we adopt the following normalization:

$$\{\mathbf{u}_i\}_{n_i}=1 \tag{8}$$

Here $\{\cdot_i\}_e$ denotes the eth component of the ith eigenvector and n_i is selected such that the n_ith component in the ith eigenvector is the largest.

II. Eigensolution Sensitivity Analysis

The system matrices $\mathbf{M}$, $\mathbf{G}(\lambda_i)$, and $\mathbf{K}$, are assumed to depend continuously on any design parameter p and their derivatives are known. Differentiating Eq. (5) with respect to p, the eigenvector derivatives satisfy

$$\mathbf{D}(\lambda_i)\frac{\partial\mathbf{u}_i}{\partial p}+\frac{\partial\mathbf{D}(s)}{\partial s}\bigg|_{s=\lambda_i}\mathbf{u}_i\frac{\partial\lambda_i}{\partial p}=-\frac{\partial\mathbf{D}(s)}{\partial p}\bigg|_{s=\lambda_i}\mathbf{u}_i \tag{9}$$

where

$$\frac{\partial\mathbf{D}(s)}{\partial p}\bigg|_{s=\lambda_i}=\lambda_i^2\frac{\partial\mathbf{M}}{\partial p}+\lambda_i\frac{\partial\mathbf{G}(s)}{\partial p}\bigg|_{s=\lambda_i}+\frac{\partial\mathbf{K}}{\partial p}$$

Equation (9) cannot be solved to find the eigenvector derivative because the matrix $\mathbf{D}(\lambda_i)$ is singular. For this reason, the normalization must be used. Differentiating Eq. (8) with respect to p yields

$$\left\{\frac{\partial\mathbf{u}_i}{\partial p}\right\}_{n_i}=0 \quad \text{or} \quad \mathbf{W}_i\frac{\partial\mathbf{u}_i}{\partial p}=0 \tag{10}$$

where

$$\mathbf{W}_i=\left\{\underbrace{0 \quad \ldots \quad 0 \quad \overbrace{\kappa}^{n_i\text{th column}} \quad 0 \quad \ldots \quad 0}_{N}\right\}$$

Here $\mathbf{W}_i$ is a $(1\times N)$ weight vector and the n_ith component of it associated with the ith right eigenvector is set to a nonzero constant κ.

Equations (9) and (10) can be rewritten as a compact linear system of algebraic equations:

$$\begin{bmatrix}\mathbf{D}(\lambda_i) & \alpha\frac{\partial\mathbf{D}(s)}{\partial s}\Big|_{s=\lambda_i}\mathbf{u}_i\\ \mathbf{W}_i & 0\end{bmatrix}\begin{Bmatrix}\frac{\partial\mathbf{u}_i}{\partial p}\\ \alpha^{-1}\frac{\partial\lambda_i}{\partial p}\end{Bmatrix}=\begin{Bmatrix}-\frac{\partial\mathbf{D}(s)}{\partial p}\Big|_{s=\lambda_i}\mathbf{u}_i\\ 0\end{Bmatrix} \tag{11}$$

where α is nonzero constants. Sometimes the components in the coefficient matrix are not all of the same order of magnitude, the nonzero constants κ and α will cause a large conditional number. Under the circumstances, the solutions of this equation are very sensitive to the effects of small errors in its coefficient matrix. To reduce the condition number, select the nonzero constant κ by finding the absolutely largest element of matrix $\mathbf{D}(\lambda_i)$ and determine the nonzero constant α by finding the largest element of matrix $\mathbf{D}(\lambda_i)$ and dividing the largest element of

$$\frac{\partial\mathbf{D}(s)}{\partial s}\Big|_{s=\lambda_i}\mathbf{u}_i$$

Substituting these constants into this system lead to a numerically well-conditioned system of algebraic equations. This system has the following form:

$$\mathbf{A}_i\mathbf{x}_i=\mathbf{b}_i \tag{12}$$

Then the derivatives of eigensolutions can be computed directly by solving the preceding algebraic system because the coefficient matrix $\mathbf{A}_i$ has a full rank matrix, as demonstrated in the following section. When $\mathbf{G}(\lambda_i)$ is constant with respect to λ_i, Eq. (12) reduces to the case of viscously damped systems. As it can be seen, this method only requires the corresponding eigenpairs information and, more important, the left eigenvectors are not required to find the eigensensitivity contrary to previous methods [3,6] for asymmetric systems with nonviscous damping. Therefore, this method is efficient in computational cost and storage capacity.

III. Numerical Stability

In this section, the coefficient matrix $\mathbf{A}_i$ will be proved to be a full rank matrix. Assume $\mathbf{A}_i\mathbf{s}=0$ for $\mathbf{s}=[\Theta \quad \tau]^T\in\mathbb{C}^{(N+1)\times 1}$. That is

$$\mathbf{D}(\lambda_i)\Theta+\alpha\frac{\partial\mathbf{D}(s)}{\partial s}\bigg|_{s=\lambda_i}\mathbf{u}_i\tau=\mathbf{0} \tag{13}$$

$$\mathbf{W}_i\Theta=0 \tag{14}$$

Premultiplying Eq. (13) by $\mathbf{v}_i^T$ yields

$$\mathbf{v}_i^T\frac{\partial\mathbf{D}(s)}{\partial s}\bigg|_{s=\lambda_i}\mathbf{u}_i\tau=0 \tag{15}$$

One can obtain $\tau=0$ for

$$\mathbf{v}_i^T\frac{\partial\mathbf{D}(s)}{\partial s}\Big|_{s=\lambda_i}\mathbf{u}_i\neq 0$$

By substituting $\tau=0$ into Eq. (13), there is

$$\mathbf{D}(\lambda_i)\Theta=\mathbf{0} \tag{16}$$

Table 1 Eigensolutions and their derivatives with respect to μ

Quantity	Elastic mode 1	Elastic mode 2	Nonviscous mode 1	Nonviscous mode 2
λ_i	$-0.2284 \pm 3.6953i$	$-1.0240 \pm 5.2878i$	-2.5350	-4.9602
$\mathbf{u}_i$	$\begin{Bmatrix} 1 \\ 0.5896 \pm 0.4413i \end{Bmatrix}$	$\begin{Bmatrix} -0.4450 \mp 0.2095i \\ 1 \end{Bmatrix}$	$\begin{Bmatrix} 1 \\ -0.7964 \end{Bmatrix}$	$\begin{Bmatrix} 0.7714 \\ 1 \end{Bmatrix}$
$\partial\lambda_i/\partial\mu$	$-1.8248 \times 10^{-2} \pm 3.2641 \times 10^{-2}i$	$-2.9285 \times 10^{-1} \pm 2.5552 \times 10^{-2}i$	-3.8383×10^{-1}	-9.9397×10^{-1}
$\partial\mathbf{u}_i/\partial\mu$	$\begin{Bmatrix} 0 \\ 4.6009 \times 10^{-2} \pm 6.3645 \times 10^{-2}i \end{Bmatrix}$	$\begin{Bmatrix} -5.2474 \times 10^{-2} \mp 3.1769 \times 10^{-2}i \\ 0 \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ -3.6253 \times 10^{-2} \end{Bmatrix}$	$\begin{Bmatrix} 7.9460 \times 10^{-4} \\ 0 \end{Bmatrix}$

Table 2 Eigensolutions at $\mu = 5.1$ and the approximated eigensolution derivatives computed by the finite difference method

Quantity	Elastic mode 1	Elastic mode 2	Nonviscous mode 1	Nonviscous mode 2
$\lambda_i(\mu = 5.1)$	$-0.2302 \pm 3.6985i$	$-1.0533 \pm 5.2901i$	-2.5734	-5.0596
$\mathbf{u}_i(\mu = 5.1)$	$\begin{Bmatrix} 1 \\ 0.5942 \pm 0.4476i \end{Bmatrix}$	$\begin{Bmatrix} -0.4502 \mp 0.2126i \\ 1 \end{Bmatrix}$	$\begin{Bmatrix} 1 \\ -7.9998 \times 10^{-1} \end{Bmatrix}$	$\begin{Bmatrix} 7.7148 \times 10^{-1} \\ 1 \end{Bmatrix}$
$\Delta\lambda_i/\Delta\mu$	$-1.7891 \times 10^{-2} \pm 3.2197 \times 10^{-2}i$	$-2.9293 \times 10^{-1} \pm 2.2729 \times 10^{-2}i$	-3.8429×10^{-1}	-9.9406×10^{-1}
$\Delta\mathbf{u}_i/\Delta\mu$	$\begin{Bmatrix} 0 \\ 4.5529 \times 10^{-2} \pm 6.2966 \times 10^{-2}i \end{Bmatrix}$	$\begin{Bmatrix} -5.2185 \times 10^{-2} \mp 3.1462 \times 10^{-2}i \\ 0 \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ -3.6304 \times 10^{-2} \end{Bmatrix}$	$\begin{Bmatrix} 7.8300 \times 10^{-4} \\ 0 \end{Bmatrix}$

As it can be seen, $\mathbf{u}_i$ is a particular solution of this equation from Eq. (5). Thus, Θ can be given by

$$\Theta = c_i \mathbf{u}_i \tag{17}$$

where c_i are constant coefficients. Substituting Eq. (17) into Eq. (14), and using normalization Eq (8), one can obtain $c_i\{\mathbf{u}_i\}_{n_i} = 0$, so $c_i = 0$ for $\{\mathbf{u}_i\}_{n_i} = 1$. Therefore, it can be concluded that $\mathbf{A}_i$ is always a full rank matrix because equation $\mathbf{A}_i\mathbf{s} = \mathbf{0}$ has the unique solution $\mathbf{s} = \mathbf{0}$.

IV. Numerical Example

To illustrate the validity of the proposed method, a simple two-degree-of-freedom asymmetric nonviscously damped system similar to [5] is considered. The equation of motion describing the free vibration of the system can be expressed by Eq. (2), with

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad G(s) = (1 + s/\mu)^{-1} \begin{bmatrix} 2 & -1.5 \\ -2.5 & 2 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} 10 & 0 \\ 0 & 20 \end{bmatrix}$$

where $\mu = 5$. The parameter μ was chosen as the real parameter p. The derivatives of eigensolutions are uniquely determined from Eq. (12) and shown in Table 1. Note that the eigenvalue derivatives calculated by the proposed method and [6] have the same results. It should be pointed out that the eigenvector derivatives computed by the proposed method and [6] are different just because the derivatives of eigenvectors adopt different constraints (As mentioned previously, the eigenvectors normalized by the two methods are different; this is the reason why the derivatives of eigenvectors are different). To illustrate the computational effort involved, a flop count comparison is made next between Adhikari's method [6] and the proposed method. For the computation of eigensolution derivatives, Adhikari's method needs to solve two linear systems of N algebraic equations; however, the proposed algorithm only needs to solve one system of $(N + 1)$ algebraic equations. Suppose that the first q eigensolution sensitivities and N_p design parameters are considered. The total number of flops to implement the proposed method is $[2N^3 + O(N_pN^2)]q$ and it is $[4N^3 + O(N_pN^2)]q$ for Adhikari's method (the flop count is considered here based on [13]). Hence, the computational effort will be reduced by about 50% in comparison with those involved by Adhikari's method. In addition, the storage capacity involved in the proposed method also will be reduced about 50% compared with that of Adhikari's method, due to the fact that Adhikari's method requires both the left and right eigenvectors.

The easiest method to implement for design sensitivity analysis may be the finite difference method [18]. The eigensolution sensitivities approximated by the first-order forward differences can be given by

$$\frac{\Delta\lambda_i}{\Delta p} = \frac{\lambda_i(p_0 + \Delta p) - \lambda_i(p_0)}{\Delta p}, \qquad \frac{\Delta\mathbf{u}_i}{\Delta p} = \frac{\mathbf{u}_i(p_0 + \Delta p) - \mathbf{u}_i(p_0)}{\Delta p}$$

Table 2 lists the approximated eigensolution derivatives computed by the finite difference method with respect to $\Delta P = 0.1$. As can be seen from Tables 1 and 2, the eigensolution derivatives computed by the proposed method show a good agreement with the results approximated by the finite difference method.

V. Conclusions

A method is proposed to calculate simultaneously and accurately the derivatives of eigensolutions for asymmetric systems with nonviscous damping. This method calculates the eigensensitivity of asymmetric systems with nonviscous damping without using the left eigenvector and requires only the eigenvector of interest. Therefore, this method is efficient in computational cost and storage capacity. Moreover, the proposed method is very compact and well

conditioned. A numerical example has demonstrated the validity of the proposed approach.

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